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Superconformal quantum mechanics of small black holes

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Abstract: Recently, Gaiotto, Strominger and Yin have proposed a holographic dual description for the near-horizon physics of certain $N = 2$ black holes in terms of the superconformal quantum mechanics on D0-branes in the attractor geometry. We provide further evidence for their proposal by applying it to the case of 'small' black holes which have vanishing horizon area in the leading supergravity approximation. We consider 2-charge black holes in type IIA on $T^2 \times M$, where M can be either K_3 or T^4, made up out of D0-branes and D4-branes wrapping M. We construct the corresponding superconformal quantum mechanics and show that the asymptotic growth of chiral primaries exactly matches with the known entropy of these black holes. The state-counting problem reduces to counting lowest Landau levels on T^2 and Dolbeault cohomology classes on M.

Keywords: AdS-CFT and dS-CFT Correspondence, Superstrings and Heterotic Strings, Black Holes in String Theory.

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Contents

1. Introduction

Compactifications of type II theory on a Calabi-Yau manifold CY_3 contain extremal black holes arising from wrapping branes around cycles of CY_3 whose near-horizon region is an $AdS_2 \times S^2 \times CY_3$ attractor geometry [1, 2]. String theory in this background is expected to be holographically dual to a conformal quantum mechanics [3], but this AdS_2/CFT_1 duality is much less well-understood than its higher-dimensional counterparts (see however [4, 5]).

However, for a class of black holes in type IIA on CY_3 carrying D0 and D4-brane charges, a concrete proposal for such a holographic dual CFT_1 was proposed by Gaiotto, Strominger and Yin (GSY) [6]. The CFT_1 takes the form of a quantum mechanics of N D0 brane probes moving in the near-horizon geometry. The super-isometry group of the background acts as a superconformal symmetry group on the quantum mechanics [7, 8]. It was proposed that the black hole ground states should be identified with the chiral primaries of this quantum mechanical system. Of particular importance were nonabelian N-D0 configurations corresponding to D2-branes wrapping the black hole horizon and carrying N units of worldvolume magnetic flux [9, 10]. These experience a magnetic flux along the Calabi-Yau directions induced by the D4-branes in the background. The chiral primary states correspond to lowest Landau levels, and their degeneracy was found to exactly reproduce the leading order entropy formula. However, the D0-D2 bound states alone did not correctly account for the known subleading corrections to the entropy formula.

The analysis of GSY was performed for 'large' black holes, which have a nonvanishing horizon area in the leading supergravity approximation. Here the D4-brane charges p^A are restricted to obey the condition

$$D \equiv \frac{1}{6} C_{ABC} p^A p^B p^C \neq 0 \,,$$

where C_{ABC} are the triple intersection numbers on CY_3. Furthermore, all p^A have to be taken to be nonvanishing and large in order for α' corrections to the background to be suppressed.

In the present work we find additional evidence for the GSY proposal by applying it to a different class of black holes which have vanishing horizon area in the leading supergravity approximation, but acquire a string scale horizon when higher derivative corrections are included [11 − 14]. For these 'small' black holes, the quantity D vanishes. We will limit ourselves to two special cases where the number of supersymmetries preserved by the background is enlarged and where the analysis becomes more tractable. We consider $D0-D4$ black holes in compactifications on $T^2 \times M$, where M can be either K_3 or T^4, and where the D4-branes are wrapped on M. These black holes are 1/2 BPS states in $N = 4$ and 1/4 BPS states in $N = 8$ supergravity respectively. In contrast to case of large black holes, only one of the magnetic charges p^A is nonzero and the D4-brane magnetic flux does not permeate all cycles in the compactification manifold but only has a component along T^2. As we shall show, this leads to a modified near-horizon behavior of the Kähler moduli of M. Placing a horizon-wrapping D2 brane with D0-brane flux in this background, we will find a quantum mechanics with a symmetry algebra that is a direct sum of an $N = 4$ superconformal algebra and the algebra of $N = 4$ supersymmetric quantum mechanics on M. The counting of chiral primaries now reduces to counting lowest Landau levels on T^2 and Dolbeault cohomology classes on M. Their asymptotic degeneracy is found to reproduce exactly the leading order entropy formula in both cases.

2. Quantum mechanics of the 2-charge black hole on $K_3 \times T^2$

2.1 Near-horizon geometry

We consider type IIA compactified on $K_3 \times T^2$ in the presence of D0-branes and D4-branes wrapped on the K_3. We choose a basis $\{\omega_A\}_{A=1\ldots 23}$ of 2-forms on $T^2 \times K_3$ in which ω_1 is the volume form on T^2 and $\{\omega_i\}_{i=2\ldots 23}$ are the 2-forms on K_3. The 4-dimensional effective theory is an $N = 4$ supergravity theory but we shall work in the $N = 2$ formalism of [15]. It contains 24 homogeneous complex scalars X^I, $I = 0\ldots 23$ and corresponding $U(1)$ gauge fields $F^I_{\mu\nu}$. Electric and magnetic charges are labelled by integers (q_I, p^I). The D0-D4 system of interest carries nonzero q_0 and p^1, with all other charges set to zero.

The prepotential, including the leading quantum correction, is given by

$$F = -\frac{1}{2} C_{ij} X^i X^j \frac{X^1}{X^0} - \frac{1}{64} \hat{A} \frac{X^1}{X^0} \,,$$

where $C_{ij} = \int_{K_3} \omega_i \wedge \omega_j$ is the intersection matrix on K_3 and $\hat{A}$ is the square of the graviphoton field strength.

As in [17], we impose a reality condition such that X^0 is real and the X^A are imaginary, so that F is also imaginary. The equations of motion for a static, spherically symmetric BPS solution carrying charges (q_I, p^I) then reduce to [16, 17]:

$$\begin{aligned}
ds^2 &= -e^{2g(r)}dt^2 + e^{-2g(r)}d\vec{x}^2\,; \qquad r \equiv \sqrt{\vec{x}^2} \\
e^{-g}(X^I - \bar{X}^I) &= i\left(h^I + \frac{p^I}{r}\right); \qquad e^{-g}(F_I - \bar{F}_I) = i\left(h_I + \frac{q_I}{r}\right) \\
\hat{A} &= -64e^{2g}(g')^2\,; \qquad e^{-K} + \frac{1}{2}\chi = -128ie^{3g}\frac{1}{r^2}\partial_r\left(r^2e^{-g}g'(F_{\hat{A}} - \bar{F}_{\hat{A}})\right) \\
F^I_{rt} &= \partial_r\left(e^g(X^I + \bar{X}^I)\right); \qquad \tilde{F}^I_{rt} = e^{2g}\partial_r\left(e^{-g}(X^I - \bar{X}^I)\right).
\end{aligned} \tag{2.1}$$

We are interested in the near-horizon limit of a solution carrying q_0 and p^1 charge. The requirement that X^0 is real imposes $q_0p^1 < 0$, and in the following we will take $q_0 < 0,\ p^1 > 0$. The constants $h^I,\ h_I$ are related to the asymptotic values of the Kähler moduli of $K_3{\times}T^2$ as $r \to \infty$. Imposing the asymptotic condition of a regular 10-dimensional geometry $M^4 \times K_3 \times T^2$ as $r \to \infty$ implies that the constants $h^I,\ h_I$ cannot all be put to zero. In our case, we are required to take h^A and h_0 to be nonzero, where $h^1 > 0$ and where the form $h^i\omega_i$ should lie in the Kähler cone of K_3.

With these asymptotic conditions one finds that in the near-horizon $r \to 0$ region, the solution to (2.1) reduces to:

$$\begin{aligned}
&e^g \simeq r\,; \qquad X^0 \simeq -\sqrt{\frac{p^1}{|q_0|}}\,; \qquad X^1 \simeq \frac{ip^1}{2}\,; \qquad X^i \simeq \frac{ih^i}{2}r \\
&\hat{A} \simeq -64\,; \qquad F^0 \simeq 2\sqrt{\frac{p^1}{|q_0|}}dt \wedge dr\,; \qquad F^1 \simeq p^1 \sin\theta d\theta \wedge d\phi\,,
\end{aligned} \tag{2.2}$$

where (θ, ϕ) are coordinates on S^2. The near-horizon metric is $AdS_2 \times S^2$. As in the discussion of [17], we expect that the full solution of (2.1) which in the near-horizon limit reduces to (2.2) does not have the behavior of flat Minkowski space with constant moduli at $r \to \infty$, but will rather contain unphysical fluctuations around it. This is an artifact of a 'bad' choice of field variables which should be removeable by making a suitable field redefinition [17, 18].

Let us briefly discuss the supersymmetry preserved by the background (2.2). It should preserve at least 8 supersymmetries, since our charge configuration is 1/2-BPS. In the near-horizon limit, some enhancement may occur, and a maximally supersymmetric background would have 16 preserved supercharges. However, this cannot be the case here since the solution does not even preserve the full subset of $N = 2$ supercharges. This would require all moduli to be constant [16] while the scalars X^i in (2.2) vary linearly with r. Therefore the preserved number of supersymmetries should be less than 16, and more than or equal to 8. Curiously, the D0 brane quantum mechanics we will write down in the next subsection will have 12 super(conformal)-charges. Presumably, this number could be understood from a detailed analysis of the $N = 4$ supersymmetry variations which we shall not attempt here.

The 10-dimensional type IIA background metric and RR fluxes corresponding to (2.2) are given by

$$\begin{aligned} ds^2 &= Q^2\left(-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \sin^2\theta d\phi^2\right) + 2dzd\bar{z} + 2rg_{a\bar{b}}dz^a d\bar{z}^{\bar{b}} \\ g_{a\bar{b}} &= \frac{ig_s}{4\pi Q}h^i(\omega_i)_{a\bar{b}}\,; \qquad F^{(4)} = \frac{p^1}{4\pi}\sin\theta d\theta d\phi\wedge\omega_1\,; \qquad F^{(2)} = \frac{Q}{g_s}dr\wedge dt\,. \end{aligned} \tag{2.3}$$

Here, we have chosen coordinates $(z,\bar{z})$ on T^2 and $(z^a,\bar{z}^{\bar{a}})_{a,\bar{a}=1,2}$ on K_3. We will work in units in which $2\pi\sqrt{\alpha'}=1$. The radius Q of $AdS_2\times S^2$ is then given by

$$Q = \frac{g_s}{2\pi}\sqrt{\frac{p^1}{|q_0|}}\,.$$

An important difference of the background (2.3) with the large black hole backgrounds of [6] is that, in the latter case, the magnetic $F^{(4)}$ flux permeates every 2-cycle on the compactification manifold, while here it only has a T^2 component. The K_3 is not supported by flux, which is why its Kähler moduli are not fixed to finite values at the horizon but vary linearly with r.

2.2 Superconformal D0-brane quantum mechanics

Following [10, 6], we will consider the quantum mechanics of a nonabelian configuration of N D0-brane probes in the background (2.3) corresponding to a D2-brane wrapping the horizon S^2. This system has an alternative description in terms of a horizon-wrapping D2-brane with N units of flux turned on on its worldvolume [9, 10]. The bosonic part of the quantum mechanics can easily be derived from the DBI and Wess-Zumino actions for the D2-brane, fixing a static gauge for the worldvolume reparametrizations. The target space seen by the brane is $R\times T^2\times K_3$ with metric

$$ds^2 = T\left(2Qd\xi^2 + \frac{\xi^2}{Q}dzd\bar{z} + \frac{1}{Q}g_{a\bar{b}}dz^a d\bar{z}^{\bar{b}}\right),$$

where we defined $\xi\equiv 1/\sqrt{r}$ and T is the mass of a horizon-wrapped D2-brane with N units of flux:

$$T = \frac{2\pi}{g_s}\sqrt{(4\pi Q^2)^2 + N^2}\,.$$

Note that the target space is in this case a direct product $R\times T^2$ and K_3. This is a consequence of the fact that the K_3 is not supported by flux and that its Kähler moduli vary linearly with r.

The bosonic hamiltonian, to quadratic order in derivatives and in the limit $N\gg Q$, reads:

$$\begin{aligned} H_{\rm bos} &= H_{\rm bos}^{R\times T^2} + H_{\rm bos}^{K3} \\ H_{\rm bos}^{R\times T^2} &= \frac{1}{8QT}P_\xi^2 + \frac{Q}{T\xi^2}(P_z - A_z)(P_{\bar{z}} - A_{\bar{z}}) + \frac{32\pi^4 Q^5}{g_s^2 T\xi^2} \\ H_{\rm bos}^{K3} &= \frac{Q}{T}P_a g^{a\bar{b}}P_{\bar{b}}\,. \end{aligned} \tag{2.4}$$

The coordinates $(z^a, \bar{z}^{\bar{a}})$ on K_3 have been chosen such that the determinant of the K_3 metric is constant. We have introduced a $U(1)$ gauge potential A on T^2 obeying $dA = 2\pi p^1\omega_1$. Explicitly, A is given by[1]

$$A = \frac{4\pi^2 iQ}{g_s}\left(\bar{z}dz - zd\bar{z}\right). \tag{2.5}$$

The dynamics on $R \times T^2$ and $K3$ decouples, a fact which, as we will see, will still be true when including fermions. This means that the symmetry group acting on the quantum mechanics will naturally split into a product $G_1 \times G_2$ with G_1 and G_2 acting on the $R\times T^2$ and K_3 parts of the wavefunction respectively. We shall show that G_1 is the $N = 4$ superconformal group $\mathrm{SU}(1,1|2)_Z$ (where Z indicates the presence of a central charge) and G_2 is the supergroup of $N = 4$ supersymmetric quantum mechanics (SQM). We shall now include the fermions and give the explicit form of the symmetry generators.

The kappa-symmetric action for a D2-brane in an arbitrary background [19] contains two sixteen-component spinors of $SO(9,1)$, one of which can be eliminated by fixing kappa-symmetry. Hence the quantum mechanics contains sixteen fermions, which are labeled as $(\lambda_\alpha, \bar{\lambda}_\alpha; \eta_\alpha, \bar{\eta}_\alpha; \eta^a_\alpha, \eta^{\bar{a}}_\alpha)_{\alpha=1,2}$. These are roughly the superpartners of the bosonic coordinates $(\xi; z, \bar{z}; z^a, \bar{z}^{\bar{a}})$. The doublet index α indicates transformation properties under an SU(2) R-symmetry which corresponds to spatial rotations. The canonical anticommutation relations for the fermions are

$$\{\lambda_\alpha, \bar{\lambda}_\beta\} = \epsilon_{\alpha\beta}\,; \qquad \{\eta_\alpha, \bar{\eta}_\beta\} = \epsilon_{\alpha\beta}\,; \qquad \{\eta^a_\alpha, \bar{\eta}^{\bar{b}}_\beta\} = \epsilon_{\alpha\beta} g^{a\bar{b}}\,. \tag{2.6}$$

The fermionic generators of the group $\mathrm{SU}(1,1|2)_Z$ acting on the $R \times T^2$ Hilbert space consist of supersymmetry generators $Q_\alpha, \bar{Q}_a$ and special supersymmetry generators $S_\alpha, \bar{S}_\alpha$ given by[2]:

$$\begin{aligned}
Q_\alpha &= \frac{1}{\sqrt{QT}}\left(\frac{1}{2}\lambda_\alpha P_\xi - \frac{i}{\xi}\eta_{(\alpha}\bar{\eta}_{\beta)}\lambda^\beta + \frac{i}{4\xi}\bar{\lambda}_\alpha\lambda^2 + \frac{i}{4\xi}\lambda_\alpha\right) + \\
&\quad + \sqrt{\frac{Q}{T}}\left(\frac{\sqrt{2}}{\xi}\eta_\alpha(P_z - A_z) - \frac{8\pi^2 Q^2}{g_s}\frac{i}{\xi}\lambda_\alpha\right) \\
\bar{Q}_\alpha &= \frac{1}{\sqrt{QT}}\left(\frac{1}{2}\bar{\lambda}_\alpha P_\xi - \frac{i}{\xi}\eta_{(\alpha}\bar{\eta}_{\beta)}\bar{\lambda}^\beta - \frac{i}{4\xi}\bar{\lambda}^2\lambda_\alpha - \frac{i}{4\xi}\bar{\lambda}_\alpha\right) + \\
&\quad + \sqrt{\frac{Q}{T}}\left(\frac{\sqrt{2}}{\xi}\bar{\eta}_\alpha(P_{\bar{z}} - A_{\bar{z}}) + \frac{8\pi^2 Q^2}{g_s}\frac{i}{\xi}\bar{\lambda}_\alpha\right) \\
S_\alpha &= 2\sqrt{QT}\xi\lambda_\alpha \\
\bar{S}_\alpha &= 2\sqrt{QT}\xi\bar{\lambda}_\alpha\,.
\end{aligned} \tag{2.7}$$

In addition to the hamiltonian $H^{R\times T^2}$, the bosonic generators of $\mathrm{SU}(1,1|2)_Z$ consist of the dilatation generator D, the generator of special conformal transformations K and the

[1]Note that our choice of coordinates on T^2 in (2.3) implies the normalization $\omega_1 = -\frac{2i}{\sqrt{|q_0|p^1}}dz\wedge d\bar{z}$.

[2]Our conventions for SU(2) index operations are "southwest-northeast", i.e. $\lambda^\alpha = \lambda_\beta\epsilon^{\beta\alpha}$, $\lambda_\alpha = \epsilon_{\alpha\beta}\lambda^\beta$, $\lambda^2 = \lambda_\alpha\lambda^\alpha$ and we take $\epsilon_{01} = \epsilon^{01} = 1$

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SU(2) R-symmetry generators $T_{\alpha\beta}$. They are given by

$$\begin{aligned} D &= \frac{1}{2}(\xi P_\xi + P_\xi \xi)\,; & K &= 2QT\xi^2 \\ T_{\alpha\beta} &= L^\lambda_{\alpha\beta} + L^\eta_{\alpha\beta}\,; & L^\lambda_{\alpha\beta} &= \lambda_{(\alpha}\bar\lambda_{\beta)};\; L^\eta_{\alpha\beta} = \eta_{(\alpha}\bar\eta_{\beta)}\,. \end{aligned} \tag{2.8}$$

The relevant anticommutation relations are

$$\begin{aligned} \{Q_\alpha, \bar Q_\beta\} &= 2\epsilon_{\alpha\beta} H^{R\times T^2}\,; \qquad \{Q_\alpha, Q_\beta\} = 0 \\ \{S_\alpha, \bar S_\beta\} &= 2\epsilon_{\alpha\beta} K\,; \qquad \{S_\alpha, S_\beta\} = 0 \\ \{Q_\alpha, \bar S_\beta\} &= \epsilon_{\alpha\beta}\left(D - \frac{16\pi^2 i Q^3}{g_s}\right) - 2iT_{\alpha\beta}\,; \{S_\alpha, \bar Q_\beta\} = \epsilon_{\alpha\beta}\left(D + \frac{16\pi^2 i Q^3}{g_s}\right) + 2iT_{\alpha\beta}\,. \end{aligned} \tag{2.9}$$

The $N = 4$ SQM acting on the K_3 Hilbert space has supersymmetry generators $\mathcal{Q}_\alpha$, $\bar{\mathcal{Q}}_\alpha$ given by

$$\begin{aligned} \mathcal{Q}_\alpha &= \sqrt{\frac{2Q}{T}}\eta^a_\alpha P_a \\ \bar{\mathcal{Q}}_\alpha &= \sqrt{\frac{2Q}{T}}\bar\eta^{\bar a}_\alpha P_{\bar a} \end{aligned} \tag{2.10}$$

with anticommutation relations

$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_\beta\} = 2\epsilon_{\alpha\beta} H^{K_3}; \qquad \{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = 0\,.$$

2.3 Counting chiral primaries and black hole entropy

Due to the existence of a dilatation generator D, the hamiltonian H (generating translations of Poincaré time) has a continuous spectrum, making the counting of its ground states ill-defined. It was therefore proposed in [6] to count instead the ground states of $L_0 = H + K$, the generator of global time translations. From (2.4), (2.8) we see that L_0 has a bound state potential and its discrete eigenstates will be localized in the radial ξ direction. The GSY proposal made in [6] states that the chiral primaries of the near-horizon D0-brane quantum mechanics are to be identified with the black hole microstates. Applied to the case at hand, this means that we have to count states of the form

$$|\psi\rangle \otimes |h\rangle\,,$$

where $|\psi\rangle$ is a chiral primary of $\mathrm{SU}(1,1|2)_Z$ and $|h\rangle$ is a supersymmetric ground state of the $N = 4$ SQM.

2.3.1 Chiral primaries of $\mathrm{SU}(1,1|2)_Z$

Chiral primaries of $\mathrm{SU}(1,1|2)_Z$ are characterized as follows [10]. We introduce the doublet notation

$$Q^{++} = Q_1\,, \qquad Q^{-+} = Q_2\,, \qquad Q^{+-} = \bar Q_1\,, \qquad Q^{--} = \bar Q_2$$

and define

$$G^{\alpha A}_{\pm\frac{1}{2}} = \frac{1}{\sqrt{2}}(Q^{\alpha A} \mp i S^{\alpha A})\,,$$

where $\alpha, A = +, -$. The anticommutation relations (2.9) now become

$$\begin{aligned}\left\{G^{\alpha A}_{\pm\frac{1}{2}}, G^{\beta B}_{\pm\frac{1}{2}}\right\} &= \epsilon^{\alpha\beta}\epsilon^{AB}L_{\pm 1} \\ \left\{G^{\alpha A}_{\frac{1}{2}}, G^{\beta B}_{-\frac{1}{2}}\right\} &= \epsilon^{\alpha\beta}\epsilon^{AB}L_0 + 2\epsilon^{AB}T^{\alpha\beta} + \epsilon^{\alpha\beta}Z^{AB}\,,\end{aligned} \tag{2.11}$$

where Z^{AB} is a c-number central charge matrix with $Z^{++} = Z^{--} = 0,\ Z^{+-} = Z^{-+} = 16\pi^2 Q^3/g_s > 0$. The second anticommutator implies a unitarity bound

$$L_0 \geq j + 16\pi^2 \frac{Q^3}{g_s} \tag{2.12}$$

with j the spin under the SU(2) R-symmetry. Primary states are annihilated by the positive moded operators $G^{\alpha A}_{\frac{1}{2}}$. Chiral primaries $\psi\rangle$ in addition saturate the bound (2.12), hence they are also annihilated by $G^{++}_{-\frac{1}{2}}$:

$$G^{\alpha A}_{\frac{1}{2}}|\psi\rangle = G^{++}_{-\frac{1}{2}}|\psi\rangle = 0\,. \tag{2.13}$$

To construct the chiral primaries we use separation of variables into an AdS_2 and a T^2 component. Denoting the T^2 component by $|\phi\rangle$, we shall see that chiral primaries are in one-to-one correspondence with states $|\phi\rangle$ satisfying

$$\eta_\alpha(P_z - A_z)|\phi\rangle = \bar{\eta}_\alpha(P_{\bar{z}} - A_{\bar{z}})|\phi\rangle = 0\,. \tag{2.14}$$

For the gauge field A given in (2.5), the equation $P_{\bar{z}} - A_{\bar{z}} = 0$ has no normalizeable solutions, while the solutions to $P_z - A_z = 0$ are the lowest Landau level wavefunctions $\phi_k(z,\bar{z})$. The number of independent lowest Landau level wavefunctions is given by an index theorem and is equal to the first Chern number [20]

$$\frac{1}{2\pi}\int_{T^2} dA = p^1\,.$$

Hence the equations (2.14) are solved by

$$|\phi_k\rangle = \phi_k(z,\bar{z})|0\rangle\,, \tag{2.15}$$

where $|0\rangle$ is the vacuum state annihilated by the $\bar{\eta}_\alpha$. These p^1 states are bosons under the SU(2) of spatial rotations.

The construction of chiral primaries from the states $|\phi_k\rangle$ now proceeds as follows. On states obtained by tensoring $|\phi_k\rangle$ with an arbitrary state in the AdS_2 part of the Hilbert space, the superconformal generators $G^{\alpha A}_{\pm\frac{1}{2}}$, $G^{++}_{-\frac{1}{2}}$ act as

$$\begin{aligned}G^{++}_{\pm\frac{1}{2}} &= \frac{\lambda^{++}}{\sqrt{2QT}}\left(\frac{1}{2}P_\xi + \frac{i}{2\xi}\lambda^{+-}\lambda^{-+} + \left(\frac{1}{4} - \frac{8\pi^2}{g_s}Q^3\right)\frac{i}{\xi} \mp 2iQT\xi\right) \\ G^{-+}_{\frac{1}{2}} &= \frac{\lambda^{-+}}{\sqrt{2QT}}\left(\frac{1}{2}P_\xi - \frac{i}{2\xi}\lambda^{--}\lambda^{++} + \left(\frac{1}{4} - \frac{8\pi^2}{g_s}Q^3\right)\frac{i}{\xi} - 2iQT\xi\right) \\ G^{+-}_{\frac{1}{2}} &= \frac{\lambda^{+-}}{\sqrt{2QT}}\left(\frac{1}{2}P_\xi + \frac{i}{2\xi}\lambda^{--}\lambda^{++} - \left(\frac{1}{4} - \frac{8\pi^2}{g_s}Q^3\right)\frac{i}{\xi} - 2iQT\xi\right) \\ G^{--}_{\frac{1}{2}} &= \frac{\lambda^{--}}{\sqrt{2QT}}\left(\frac{1}{2}P_\xi - \frac{i}{2\xi}\lambda^{+-}\lambda^{-+} - \left(\frac{1}{4} - \frac{8\pi^2}{g_s}Q^3\right)\frac{i}{\xi} - 2iQT\xi\right)\,.\end{aligned} \tag{2.16}$$

Normalizeable states annihilated by $G^{++}_{-\frac{1}{2}}$ have to be annihilated by λ^{++}. Such states automatically also have $G^{++}_{\frac{1}{2}} = 0$. If we choose the states to be annihilated by λ^{-+} as well, they will have $G^{-+}_{\frac{1}{2}} = 0$ while the equations $G^{\pm-}_{\frac{1}{2}} = 0$ lead to a single differential equation for the ξ-part of the wavefunction.[3] The resulting chiral primary states are given by

$$|\psi_k\rangle = \xi^{-\frac{1}{2}+\frac{16\pi^2 Q^3}{g_s}} e^{-2QT\xi^2}|\bar{0}\rangle \otimes |\phi_k\rangle\,, \tag{2.17}$$

where $|\bar{0}\rangle$ is annihilated by λ^{++} and λ^{-+} and hence is bosonic under the rotational SU(2). We have constructed in this manner p^1 bosonic chiral primary states of $\mathrm{SU}(1,1|2)_Z$. It's also possible to show that with the states (2.17) we have found all chiral primaries.

2.3.2 N=4 supersymmetric ground states

A supersymmetric ground state in the $N = 4$ supersymmetric quantum mechanics satisfies

$$\mathcal{Q}_\alpha|h\rangle = \bar{\mathcal{Q}}_\alpha|h\rangle = 0\,.$$

Such states are well-known to be in one-to-one correspondence with the Dolbeault cohomology classes on K_3 (see e.g. [21]). This can be seen by representing states $|h\rangle$ by differential forms on K_3 and identifying

$$\begin{aligned} \eta_1^a &\leftrightarrow dz^a\,, & \bar{\eta}_1^{\bar{a}} &\leftrightarrow -d\bar{z}^{\bar{b}} \\ \eta_2^a &\leftrightarrow g^{a\bar{b}}\frac{\delta}{\delta(d\bar{z}^{\bar{b}})}\,, & \bar{\eta}_2^{\bar{a}} &\leftrightarrow g^{\bar{a}b}\frac{\delta}{\delta(dz^b)} \end{aligned} \tag{2.18}$$

so that the anticommutation relations (2.6) are satisfied. Under this identification, bosonic and fermionic states are represented by even and odd forms respectively. The supersymmetry generators are (up to proportionality constants) identified with the Dolbeault operators

$$\begin{aligned} \mathcal{Q}_1 &\leftrightarrow \partial & \bar{\mathcal{Q}}_1 &\leftrightarrow \bar{\partial} \\ \mathcal{Q}_2 &\leftrightarrow \bar{\partial}^\dagger & \bar{\mathcal{Q}}_2 &\leftrightarrow \partial^\dagger\,. \end{aligned} \tag{2.19}$$

Since K_3 has 24 even harmonic forms, we find 24 bosonic ground states of the $N = 4$ SQM.

2.3.3 Black hole entropy

Tensoring together the chiral primaries of $\mathrm{SU}(1,1|2)_Z$ and the supersymmetric ground states of the N=4 SQM we find in total $24p^1$ bosonic chiral primaries. Note that this number does not depend on the background D0-brane charge $|q_0|$; hence for the purpose of counting ground states we can take $q_0 \to 0$ and count the number of chiral primaries with total D0-brane charge N in a background with fixed magnetic D4-charge p^1. The large degeneracy of such states comes from the many ways the total number N of D0-branes can

[3] Equivalently, one could proceed as in [6] and start instead from the vacuum annihilated by λ^{++} and λ^{+-}. Adjusting the wavefunction one can construct states that are annihilated by $G^{++}_{\pm\frac{1}{2}}, G^{\pm-}_{\frac{1}{2}}$ but not by $G^{-+}_{\frac{1}{2}}$. Acting with $G^{-+}_{\frac{1}{2}}$ on these states one obtains chiral primaries. This construction leads to the same states (2.17).

be split into k smaller clusters of n_i D0's such that

$$\sum_{i=1}^{k} n_i = N\,,$$

each cluster corresponding to a wrapped D2-brane that can reside in any of the $24p^1$ bosonic chiral primary states. The counting problem is the same as the counting the degeneracy d_N of states at level N in a $1+1$ dimensional CFT with $24p^1$ bosons. The generating function is then

$$\begin{aligned} Z &\equiv \sum d_n q^n \\ &= \mathrm{Tr}\, q^N \\ &= \prod_n (1-q^n)^{-24p^1}\,. \end{aligned} \tag{2.20}$$

This gives the asymptotic degeneracy at large N

$$\ln d_N \approx 4\pi\sqrt{Np^1}$$

which indeed corresponds to the known entropy of the black hole obtained either from microscopic computation [22 − 24] or from the supergravity description incorporating higher derivative corrections [25, 13, 14, 29, 17, 26 − 28].

3. Quantum mechanics of the 2-charge black hole on T^6

We will now consider a compactification on T^6 with the same charge configuration as before, i.e. a background produced by q_0 D0-branes and p^1 D4-branes wrapping a T^4. These are 1/4 BPS black holes of 4-dimensional $N = 8$ supergravity and hence preserve the same number of supersymmetries as the 2-charge black hole on $T^2 \times K_3$. These black holes also have a vanishing horizon area in the leading approximation, and the arguments of [12] show that, in this case also, a horizon is generated when including higher derivative corrections.

The situation is however less clear-cut than before, as the corrections to the prepotential vanish in this case. Hence the corrections that generate the horizon are expected to come from non-holomorphic corrections to the supergravity equations, and it is not known how to incorporate these systematically at present. In this section, we shall be a little cavalier and simply assume that the near-horizon limit of the quantum corrected background is still of the form (2.3), with the K_3 metric now replaced by the metric on T^4 and possibly with a different value of the constant Q.

The resulting quantum mechanics of a wrapped D2-brane with D0-brane flux then reduces to (2.4), (2.9), (2.10) with $g_{a\bar{b}}$ replaced by the flat metric on T^4. The symmetry group again splits into a superconformal $SU(1, 1|2)_Z$ acting on the $R \times T^2$ part of the Hilbert space and an $N = 4$ supersymmetric quantum mechanics acting on the T^4 part. The counting of chiral primaries of $SU(1, 1|2)_Z$ goes trough as in section 2.3, yielding p^1 bosonic chiral primaries. The difference lies in the counting of ground states of the $N = 4$ SQM, now corresponding to the Dolbeault cohomology of T^4. Since T^4 has 8 even and

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8 odd harmonic forms, we find 8 bosonic and 8 fermionic ground states. The counting problem for chiral primaries is now isomorphic to counting the degeneracy at level N of a CFT with $8p^1$ bosons and $8p^1$ fermions. The partition function is

$$Z = \prod_n \left(\frac{1+q^n}{1-q^n}\right)^{8p^1} . \tag{3.1}$$

This gives the asymptotic degeneracy

$$\ln d_N \approx 2\sqrt{2}\pi\sqrt{Np^1}$$

which is in agreement with the known degeneracy from microscopic counting [22 – 24].

4. Discussion

In this paper we provided additional evidence for the GSY proposal by counting the asymptotic degeneracy of chiral primaries in the superconformal quantum mechanics describing the near horizon physics of small black holes and showing it to agree with the black hole entropy. We now list some open questions.

- Our construction of the quantum mechanical superconformal symmetry algebra suggests that the number of supercharges in the near-horizon background (2.2) is enhanced from 8 in the bulk to 12. It would be interesting to prove this directly from the $N = 4$ supersymmetry variations.
- In section 3 we assumed the near-horizon geometry of the 2-charge black hole on T^6 including quantum corrections to be of the form (2.3), and saw that this led to the correct counting of chiral primaries. The quantum corrections are expected to come from non-holomorphic corrections to the supergravity equations and it would be of interest to check whether known nonholomorphic corrections such as R^4 terms indeed lead to (2.3).
- It is not clear in how far the agreement found in this work depended on the large number of 8 supersymmetries preserved by the small black holes considered here. In particular, it would be interesting to verify whether for small black holes which are 1/2 BPS states in an $N = 2$ compactification, preserving only 4 supersymmetries, the counting of chiral primaries in the D0-D2 quantum mechanics still reproduces the correct entropy formula.
- The small black holes considered here could also prove to be a good testing ground for verifying or refining the GSY proposal in order to reproduce the correct subleading corrections to the entropy formula. It should be noted that, although the chiral primary partition functions (2.20), (3.1) only reproduce the leading term in the entropy formula, a modification of the counting problem would produce the microcanonical partition function to all orders in both cases. Whether such a modification can be justified in this context remains to be seen.

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